Band structures of one-dimensional subwavelength photonic crystals containing metamaterials

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We study the band structures of one-dimensional (1D) photonic crystals (PCs) in the subwavelength limit. A "spatial-averaged single-negative" (SASN) gap whose edges correspond to zero (volume) averaged permittivity ($\bar{\varepsilon}=0$) and zero (volume) averaged permeability ($\bar{\mu}=0$) will appear when metamaterial is included in the PC. Unlike the Bragg gap, the frequency range of the SASN gap is invariant to the geometrical scaling and insensitive to the incident angle and disorder. In the subwavelength limit, both the zero-$\bar{n}$ gap in the left-handed 1D PC and the zero-effective-phase gap in the single-negative 1D PC can be understood as SASN gaps. When the subwavelength condition is not fulfilled, the zero-$\bar{n}$ gap and zero-effective-phase gap begin to act differently.

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Conventional photonic band gap (PBG) materials are a type of artificial composites with periodically modulated dielectric function, and the photonic gaps are a consequence of Bragg scattering in these materials [1]. Therefore, the PBG frequency is inversely proportional to the lattice constant, and also, randomness will destroy the band gap [2]. Recently left-handed materials (LHMs) with negative permittivity and negative permeability, which were first suggested theoretically by Veselago [3], have attracted much attention recently, thanks to the experimental realization of such materials (though anisotropic) at microwave frequencies [4] and the theoretical awareness of using a LHM slab as a perfect lens that will produce an image of any object with perfect resolution [5]. Isotropic LHMs have been introduced, too [6].

One-dimensional (1D) photonic crystal (PC) structures consisting of alternating LHM and right-handed material (RHM) layers have already been investigated through calculating the transmittance or the reflectance of the structures [7]. The effects of photon tunneling and reflective Bragg region were observed. In Ref. [8], a more interesting phenomenon is predicted by analysis as well as by numerical simulation: when the volume average of the effective refractive index equals zero, a new type of gap, named the zero-$\bar{n}$ gap, emerges. Unlike conventional Bragg gaps, the zero-$\bar{n}$ gap is invariant to the geometrical scaling of the superlattice and insensitive to the wave polarization, angle of incidence, or the period of the structure [8–11]. In a Fibonacci sequence consisting of slabs of LHM and RHM, band structure also shows that the zero-$\bar{n}$ gap is less sensitive to the incident angle and the polarization than the Bragg gap [12]. Recently, the zero-$\bar{n}$ gap was verified experimentally by measuring the scattering parameters of a 1D layered stack composed of LHM and RHM [13]. Moreover, it is found that typical broad Bragg reflectivity peaks appearing in 1D LHM-RHM PCs are very narrow [8,14].

Besides the LHMs, materials with only one negative material parameter have attracted interest [15,16]. Single-negative (SNG) materials include the ϵ-negative (ENG) media with negative permittivity and positive permeability and the μ-negative (MNG) media with negative permeability and positive permittivity. A number of unique properties such as resonance, complete tunneling, and transparency have been found in MNG-ENG bilayer structure [16]. The transmission properties of a 1D PC containing both kinds of SNG medium shows the possession of a PBG with zero effective phase [17,18]. The omnidirectional zero-effective-phase gap is insensitive to incident angles or light polarizations and invariant upon a change of scale length [18]. Such omnidirectional gap results from the interaction of evanescent waves.

As we know, the physical origination of the zero-$\bar{n}$ gap in LHM-RHM PCs and the zero-effective-phase gap in MNG-ENG PCs are quite different. In LHM-RHM PCs, the interaction of propagating waves causes the formation of PBGs. When the supercell unit is of subwavelength size, the zero-$\bar{n}$ gap originates mainly from the phase-shift cancellation in every supercell unit, which differs itself from Bragg gaps. When the supercell is much larger, the long-range interference becomes the dominant factor in the formation of the zero-$\bar{n}$ gap. In contrast, PBGs in MNG-ENG PCs originate from the interaction of evanescent waves. However, as is mentioned above, two gaps show many similar properties. In what follows, we focus our attention on understanding and explaining this interesting phenomenon.

The 1D PC we are about to work on is periodic along the longitudinal direction z, with $a=d_1+d_2$ being the corresponding spatial periodicity. Here, d_1 and d_2 are the thicknesses of the two slabs contained in the primary unit supercell corresponding to the materials with ε_1,μ_1 and ε_2,μ_2, respectively. Assuming that the electromagnetic field in the supercell is a Bloch wave, i.e., $F(z+a)=e^{ika}F(z)$, one can easily derive the dispersion relationship of the photonic modes [19]:

$$\cos[k(d_1+d_2)]=\cos(\alpha_1 d_1)\cos(\alpha_2 d_2)-\frac{1}{2}\left(\frac{F_2}{F_1}+\frac{F_1}{F_2}\right)\sin(\alpha_1 d_1)\sin(\alpha_2 d_2), \quad (1)$$

with

$$\alpha_i=\frac{\omega\sqrt{\varepsilon_i}\sqrt{\mu_i}}{C}\cos\theta_i, \quad F_i=\sqrt{\frac{\varepsilon_i}{\mu_i}}\cos\theta_i,$$

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$$\cos\theta_i = \sqrt{1 - \frac{1}{\varepsilon_i \mu_i}\left(\frac{k_\parallel c}{\omega}\right)} \quad (i = 1,2)$$

for *s*-polarized waves and

$$\alpha_i = \frac{\omega\sqrt{\varepsilon_i}\sqrt{\mu_i}}{C}\cos\theta_i, \quad F_i = \sqrt{\frac{\mu_i}{\varepsilon_i}}\cos\theta_i,$$

$$\cos\theta_i = \sqrt{1 - \frac{1}{\varepsilon_i \mu_i}\left(\frac{k_\parallel c}{\omega}\right)} \quad (i = 1,2)$$

for *p*-polarized waves. Here, $\theta_{1,2}$ are the angles between the propagating waves and the normal to the interface in the two media, respectively. $k_\parallel$ is the parallel part of the wave vector. If the absolute value of the right-hand side of Eq. (1) is larger than 1, the corresponding solution will have a nonzero imaginary part, which means that the 1D PC does not support propagating Bloch modes.

In the subwavelength limit, i.e., $\alpha_i d_i \ll 1$, one can expand $\cos(\alpha_i d_i)$ and $\sin(\alpha_i d_i)$ in a Taylor series,

$$\cos(\alpha_i d_i) = 1 - \frac{(\alpha_i d_i)^2}{2} + O((\alpha_i d_i)^2),$$

$$\sin(\alpha_i d_i) = \alpha_1 d_1 + O(\alpha_i d_i). \tag{2}$$

Substituting Eq. (2) into Eq. (1) and neglecting the high-order terms, we have the dispersion relationship simplified as

$$\cos[k(d_1 + d_2)] = 1 - \frac{(\alpha_1 d_1)^2}{2} - \frac{(\alpha_2 d_2)^2}{2} - \frac{1}{2}\left(\frac{F_2}{F_1} + \frac{F_1}{F_2}\right)\alpha_1 d_1 \alpha_2 d_2. \tag{3}$$

It is obvious that the band gap occurs at the position where $\cos[k(d_1+d_2)]=1$. The gap edges appear as

$$\frac{(\alpha_1 d_1)^2}{2} + \frac{(\alpha_2 d_2)^2}{2} + \frac{1}{2}\left(\frac{F_2}{F_1} + \frac{F_1}{F_2}\right)\alpha_1 d_1 \alpha_2 d_2 = 0. \tag{4}$$

The answer is

$$\varepsilon_1(\omega)d_1 + \varepsilon_2(\omega)d_2 - \frac{ck_\parallel}{\omega}\left(\frac{d_1}{\mu_1(\omega)} + \frac{d_2}{\mu_2(\omega)}\right) = 0$$

or

$$\mu_1(\omega)d_1 + \mu_2(\omega)d_2 = 0 \tag{5a}$$

for *s*-polarized waves and

$$\varepsilon_1(\omega)d_1 + \varepsilon_2(\omega)d_2 = 0$$

or

$$\mu_1(\omega)d_1 + \mu_2(\omega)d_2 - \frac{ck_\parallel}{\omega}\left(\frac{d_1}{\varepsilon_1(\omega)} + \frac{d_2}{\varepsilon_2(\omega)}\right) = 0 \tag{5b}$$

for *p*-polarized waves.

Equations (5a) and (5b) imply that a transmission gap appears only if both the permeability and the permittivity change signs across the interfaces of the two mediums. That is to say, the structure must be an LHM-RHM or a ENG-MNG superlattice but not a RHM-RHM nor an LHM-LHM superlattice. In consideration of causality [20], we define the dispersive permeability and permittivity of the LHM in the LHM-RHM superlattice as [10]

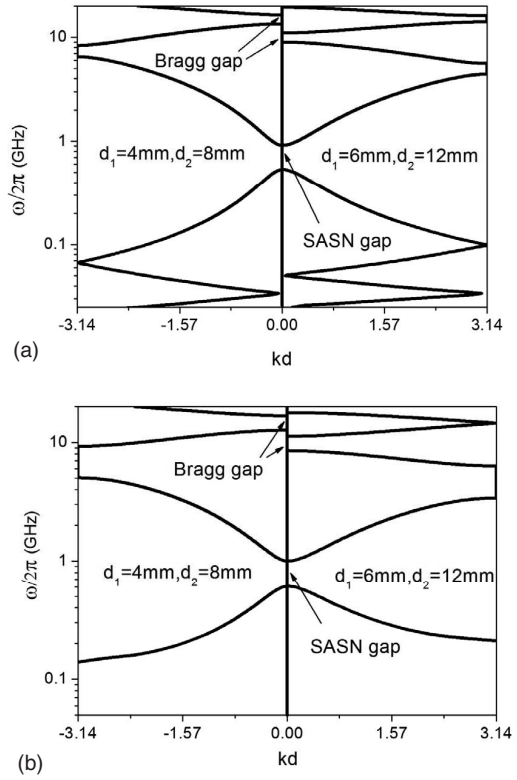

FIG. 1. The band structures at normal incidence for different cell sizes with the same $d_1/d_2=0.5$. (a) LHM-RHM PC, $\varepsilon_{10}=\mu_{10}=\mu_2=1$, $\varepsilon_2=4$, $\alpha=\beta=100$; (b) ENG-MNG PC, $\varepsilon_{20}=\mu_{10}=1$, $\varepsilon_1=\mu_2=3$, $\alpha=\beta=100$.

$$\varepsilon_1(\omega) = \varepsilon_{10} - \frac{\alpha}{\omega^2}, \quad \mu_1(\omega) = \mu_{10} - \frac{\beta}{\omega^2}. \tag{6}$$

For the ENG-MNG superlattice, we set

$$\varepsilon_1 = \varepsilon_{10}, \quad \mu_1(\omega) = \mu_{10} - \frac{\alpha}{\omega^2} \tag{7a}$$

in MNG materials and

$$\varepsilon_2(\omega) = \varepsilon_{20} - \frac{\beta}{\omega^2}, \quad \mu_2 = \mu_{20} \tag{7b}$$

in ENG materials.

For normal incidence, i.e., $k_\parallel=0$, it is obvious that the ratio of d_1 to d_2 is what really matters for the gap edges. The actual geometrical size of the whole structure and the lattice constant does not affect the position of the gap edges (see Fig. 1). One thing to note is that this special gap is the only gap exhibiting this characteristic. The gaps in the higher- or lower-frequency domain behave as Bragg gaps do. In addition, from effective medium theory, we know that this gap appears in the frequency region where the whole structure is

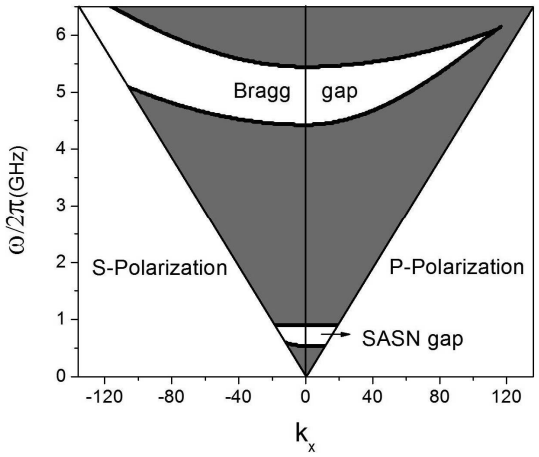

FIG. 2. The band structure of LHM-RHM PCs for different $k_{\|}$ within the cone of light. The parameters are $\varepsilon_{10}=1.21$, $\mu_{10}=1$, $\varepsilon_2=4$, $\mu_2=1$, $\alpha=\beta=100$, $d_1=0.006$ m, and $d_2=0.012$ m.

a spatial-averaged single-negative (SASN) structure; as such, it is natural to see that the wave does not propagate in it.

For oblique incidence, we find that, for the SASN gap, one gap edge is invariant with the change of $k_{\|}$ while the other is insensitive to this change. In contrast to the Bragg gaps, $k_{\|}$ for the SASN gap is limited by the light cone in a small range around zero; thus, the impact of $k_{\|}$ in Eqs. (5a) and (5b) is relatively weak. As is shown in Fig. 2, the band structure is plotted within the light cone, i.e., $k_{\|}=\omega/c$. In contrast, the Bragg gap shows a large dependence on both $k_{\|}$ (i.e., the incident angle) and polarization.

Furthermore, the SASN gap shows insensitivity to disorder, which can be explained by Eqs. (5a) and (5b), too. Keeping the ratio of the thickness of the two neighboring slabs constant, the position of the gap does not move even if the periodicity is broken.

We demonstrate by simple calculation for normal incidence that the gap edges of the zero-$\bar{n}$ gap in LHM-RHM structure [8,10] satisfy Eqs. (5a) and (5b) well. In Ref. [8], the author explained that, according to the dispersion relationship, when zero $\bar{n}$ [i.e., $\sqrt{\varepsilon_1(\omega)}\sqrt{\mu_1}d_1+\sqrt{\varepsilon_2(\omega)}\sqrt{\mu_2}d_2=0$] is achieved, there is no real solution for the wave vector and thus a PBG will be opened. That explains why this kind of gap is named the zero-$\bar{n}$ gap. It now raises a question for us whether the SASN gap decided by Eqs. (5a) and (5b) is exactly the same as the zero-$\bar{n}$ gap or whether we can tell the two gaps apart simply by changing the parameters. In general, the frequency dependence of the effective permeability and permittivity [e.g., Eq. (6) in this paper, Eq. (13) in Ref. [8]] is monotonically increasing, so it is easy to see that the frequency for zero $\bar{n}$ is between the frequencies for zero $\bar{\varepsilon}$ and zero $\bar{\mu}$. Therefore, it will be impossible to tell the two gaps apart.

We also compare gap edges of the zero-effective-phase gap in Refs. [17,18] with the SASN gap-edge condition at normal incidence and find that they satisfy Eqs. (5a) and (5b), too. The consistency can also be verified by the gap-edge condition in an equivalent transmission-line model [Eq. (6) in Ref. [17]].

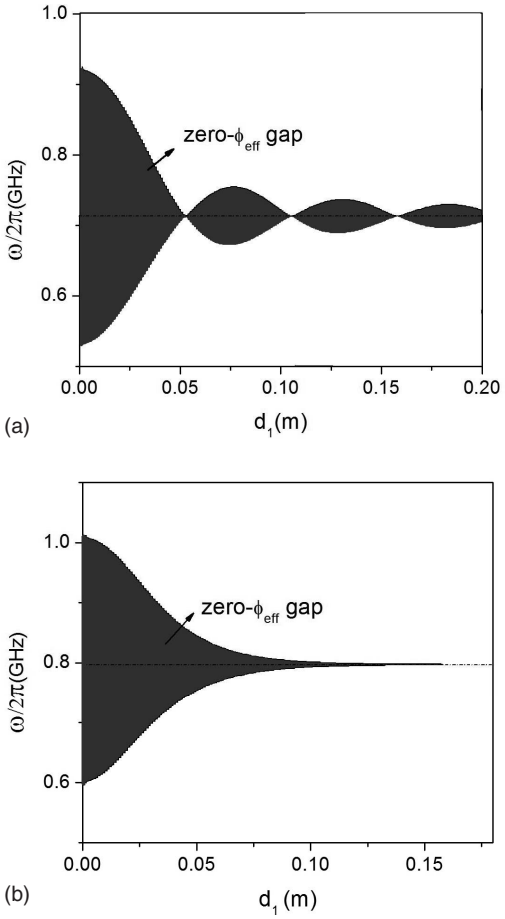

FIG. 3. The variance of the zero-$\bar{n}$ gap and the zero-effective-phase gap with scaling. The ratio of d_1 to d_2 is kept as constant: $d_1/d_2=0.5$. (a) Zero-$\bar{n}$ gap in LHM-RHM PCs. The material parameters are the same as Fig. 1(a). (b) Zero-effective-phase gap in ENG-MNG PCs. The material parameters are the same as Fig. 1(b).

The zero-effective-phase gap in the ENG-MNG structure and the zero-$\bar{n}$ gap in the LHM-RHM structure should be considered as the same kind of PBGs, since their gap-edge conditions satisfy the same parameter equations. This sounds ridiculous, but actually, it is understandable. Fredkin and Ron [15] have shown that a layered structure with alternating slabs of ENG and MNG materials may effectively behave as an LHM, because the effective group velocity in such a structure would be antiparallel with the effective phase velocity. The behavior of the wave propagation within the two structures is different, but for a short range of distance, the difference is negligible, since the trigonometric sinusoidal and hyperbolic sinusoidal functions are similar in a small-argument approximation. One can then anticipate a "thin" ENG-MNG pair to function like a thin LHM-RHM pair. This equivalence has been demonstrated by transmission-line models [21]. Propagating waves are absent in each layer since the wave vectors are complex, but in the whole periodic structure propagation modes still exist. The appearance of propagation modes can be explained with the aid of a tight-binding model in solid-state physics. When SNG layers construct a periodic structure, the localized interface modes in each period will interact and thus split. In other words, the

interface modes will couple each other and form propagation modes.

However, our SASN condition [Eqs. (5a) and (5b)] is only valid in the subwavelength limit, seeing that the effective medium theory will lose its validity and the concept of a SASN gap becomes meaningless when the subwavelength condition is not fulfilled. Keeping d_1/d_2 constant and increasing the geometrical size of the cell, we find in Fig. 3(a) that the zero-$\bar{n}$ gap in LHM-RHM superlattices narrows down gradually with some small oscillations to the central frequency corresponding to zero $\bar{n}$. The gap width becomes sensitive to scaling when the cell is large enough. Closures of the gap appear when the optical path length of each slab equals any integer multiples of a half-wavelength. As for the SNG superlattice [see Fig. 3(b)], the gap shrinks to a certain frequency monotonically. The oscillations in Fig. 3(a) originate from the Bragg interference of the propagating waves, while in Fig. 3(b) they are absent since there are only evanescent waves.

In conclusion, in the subwavelength limit, we find an SASN gap appearing in the 1D PC constructed by LHM-RHM or ENG-MNG superlattice structure. The SASN gap is invariant to scaling and insensitive to disorder or incident angle. These unusual properties offer a potential for new devices such as highly directive sources, wave front converters, or delay lines with zero phase difference between the input and output ports [22,23]. The discovery of the analytical parameter relationship of SASN gap edges sheds light on more convenience and accuracy in tuning the bandwidth. When the subwavelength condition is no longer satisfied, two gaps begin to behave differently owing to the different wave modes in different structures. These unusual properties will disappear in that case.

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